T7 Frequency and phase shift measurement

# **Measure object:** Wien circuit

**Purpose:** Determination of the phase characteristics of the circuit - ϕ(f)

**Method:** digital oscilloscope.

# **Properties of Wien bridge**



Fig.1. Wien circuit scheme. Characteristics of the circuit: amplitude and phase.

Wien circuit is a resonant circuit with frequency characteristics corresponding to a selective filter. The amplitude characteristic at the resonance frequency has a resonant peak with a voltage attenuation factor of 1/3. And the phase characteristic for this frequency goes through zero.

Thanks to the mentioned property, the circuit has been widely used in low-frequency generation systems and selective filters.

##### Measurement method description

Measurements of the phase characteristics using oscilloscope can be done in two ways, using the following:

1. images of time curves of compared voltages,

2. Lissajous curves, and the measurement is called the "ellipse method".

Ad. 1. The method of measuring the phase shift ϕ is illustrated in Figure 2.



Fig 2. Time curves method

The diagrams of the two compared voltages are shifted relative to each other by time τ. Referring τ to their period T, a phase shift determined in radians is obtained. The dependence should be used to calculate ϕ in degrees.

ϕ o =360o (τ / T).

When half of the period is read from the waveform image, the phase shift should be determined from the formula

# ϕ o =180o (τ / tT/2).

# **Ad. 2.** To measure ϕ using the ellipse method, the **"X-Y" function** is used in the oscilloscope. Then, the voltages applied to both inputs of the oscilloscope affect the individual pairs of the oscilloscope lamp deflection plates, and the result is the creation of an ellipse-shaped image on the screen (in particular, it may be a straight line or a circle). The phase shift is determined by reading the length of the characteristic sections of the ellipse (Figure 3), using the formula

ϕ o =(180o/π) arc sin(a/b).



Fig. 3. Lissajoux curves method

# **Measurement technique**

Measurements of the oscilloscope rely on reading the length of the relevant sections of the analyzed waveform from the screen. Therefore, the waveform image should be formed in such a way that readings errors are not too high. This will happen when a still and clear image is obtained and large enough to make the read sections as long as possible.

In the measurement of phase shifts by the method of image analysis of time courses, first determine the centre position of both lines of the time base, i.e. on the level of the main axis of the screen line grid. Amplitudes of the compared voltages should cover the whole height of the screen, and in its full width should be included one period of transitions. The reading error can be significantly reduced when the waveform image is only half a period for measurements - because then, the segment lengths read are twice as long as the full period image readings.

In measuring the phase shift using the ellipse method, first determine the central position of the light spot on the screen, which is achieved by two positioning knobs of the image, with both inputs closed to ground (GND position of the switch). Then create an ellipse image that covers as much of the screen as possible. The readings of sections a and b should be made after the vertical positioning of the ellipse in relation to the selected baseline.

#### Measurement tables

**Tab. 1. Measurement of an phase shift (from the image of time courses)**

|  |  |  |
| --- | --- | --- |
| **f**kHz | Readings  | Measurement results |
| **l**τdz | lT/2dz | **ϕ** [ 0 ] | **ϕ ± U(ϕ)**p=0,95 | **Ur(ϕ)**% |
| ................................................................................................f0= .......................................................................................................... |  |  |  |  |  |

**Tab. 2. Measurement of an phase shift (elipse method)**

|  |  |  |
| --- | --- | --- |
| **f**kHz | Readings  | Measurement results |
| **l**a**dz** | lb**dz** | **ϕ** [ 0 ] | **ϕ ± U(ϕ)**p=0,95 | **Ur(ϕ)**% |
| ..............................................................................................f0= ...................................................................................................... |  |  |  |  |  |

**Tab 3. Voltages and phase shift directly from oscilloscope**

|  |  |  |
| --- | --- | --- |
| **f**kHz | Readings  | Calculations |
| **Uinput****V** | **Uoutput****V** | **ϕ [ 0 ]** | **k=Uoutput/Uinput** | **Ur(ϕ)****%** |
| ..............................................................................................f0= ...................................................................................................... |  |  |  |  |  |

**Analysis of** **measurement uncertainty**

In the discussed measurements, the dominant sources of errors are the length readings: lτ , lT/2, la and lb. The grid of the oscilloscope screen line allows for length measurements with a 0.1 or 0.2 plot error rate.

On the other hand, the accuracy of the oscilloscope does not significantly affect the measurement results, because they result in the calculation of the ratio of the two lengths, measured with the invariant setting of appropriate time and gain coefficients (cu, ct). Hence, these coefficients do not appear in the equations of both methods of ϕ measurement.

The method of measuring time segments lτ  and lT/2 (Time curves method)

From formula: , can be determined relative total standard uncertainty

 ,

where:  ; ;

Δolτ, ΔolT/2 – reading errors of segment lenghts lτ and lT/2 (0,1 or 0,2 div)

Relative expanded uncertainty: 

Expanded uncertainty:

Elipse method (Lissajous curves method)

 Measurement equation: 

 *coefficient  should be omitted if the result of the angle calculation has the dimension in degrees.*

Total standard uncertainty:

,

where are errors of lenght la and lb readings.

Knowing that the function y = arc sinx has derivative , this above relation can lead to a form convenient in the calculation, substituting formulas:

, 



 After calculations we have:



Expanded uncertainties:



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Fig. 4. Measurement circuit

Comments on the performance report

The report should include drawings of the measured characteristics - both on one graph. Also, evaluate the usefulness of the oscilloscope for phase shift measurements. Compare both methods in terms of accuracy and convenience of measurements.

It is obvious that the readings of small section lengths are burdened with very large errors, i.e. the oscilloscope is not useful for measuring small phase shifts. Assuming a maximum permissible uncertainty of measurements of no more than 10%, the values of the phase shift angles from which the oscilloscope measurements are practically useful should be estimated.